Teachers Exchange in Korea (Housing Market with Contracts)

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Introduction

- In Korea, elementary and middle school teachers are (semi) public officers.
    - Cannot freely change where to work.
    - Especially, there are regulations to move across states.

Introduction

- There are exchange program for the teachers who are in need
  - spouse moved to another state
  - have parents to be taken care of

- Currently, they can apply for an exchange of ONLY one term
  - Temporal exchange (2 years)
  - Permanent swap

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  - have parents to be taken care of
- Currently, they can apply for an exchange of ONLY one term
  - Temporal exchange (2 years)
  - Permanent swap
- Exchanges are made in 1:1 between states.
  - A teacher in state S wants to move to state B. To make an exchange, there should be another teacher in B who wants to move to S, and also the two should have applied for the same term exchange.

Introduction

- There is no centralized mechanism for this, but the process is governed by each state's Office of Education.
  - Teachers submit applications to their states' Office of Education.
  - Each state assigns priorities to applicants.
  - States review potential swapping teachers' documents.
  - If there is no problem, swapping is approved.

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  - States review potential swapping teachers' documents.
  - If there is no problem, swapping is approved.
- Order of swapping is not relevant as a teacher can submit only one application.

Data

- No overall data available
- Only scattered information in each Office of Education homepage
- Worse, almost no final matching data (even the number of matches.)

Data

- Middle school applicants for 2019 exchange
  - Busan to Seoul (39) / total (137)
  - Seoul to Busan (9) / total (74)

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  - Busan to Seoul (39) / total (137)
  - Seoul to Busan (9) / total (74)
- In 2019 Gyangju to:
  - Seoul (3) Daejeon (1) Gyunggy (3) Jeonnam (1) Sejong (3)

How to Improve?

- Two possible improvements
  - Combining two terms into one application (with centralized mechanism)
  - Allowing 3-way or more exchanges (Roth, Sönmez, and Ünver.

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  - Allowing 3-way or more exchanges (Roth, Sönmez, and Ünver.
- One state per teacher restriction remains.

Model

As an extension of the **housing market** (Shapley and Scarf, 1974), a **housing market with contract** is a tuple $(I, H, X, \succ)$, where

I = finite set of n agents,

H = finite set of n houses where agent i owns house h_i,

X = a set of contracts that specifies an agent $i(x)$, a house $h(x)$, and the contractual term $t(x)$,

where $t = \{S, L\}$,

$\succ$ = a set of agents' strict preference over contracts.

Model

Definition

A matching μ is **feasible** if,

- $|\mu(i)| = 1$ for all $i \in I$,
- $i \neq j$ implies $h(\mu(i)) \neq h(\mu(j))$.

Definition

A matching μ is **individually rational** if each agent is matched to a contract that is weakly preferred to remaining unmatched i.e., for every i,

$$\mu(i) \succeq_i \bar{i}$$

, where $\bar{i}$ is an option of remaining unmatched.

Model

Definition

A matching μ is **equal-term** if each agent is matched to a contract that has the same term that her house is matched to, i.e., for every i,

$$t(\mu(i)) = t(\{x|h(x) = h_i\})$$

Model

Definition

A matching μ is **Pareto efficient** if there is no matching ν such that $\nu(i) \succeq_i \mu(i)$ for all $i \in I$ and $\nu(i) \succ_i \mu(i)$ for some $i \in I$.

Model

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Definition

A matching μ is **equal-term efficient** if, μ is equal-term and there is no equal-term matching ν such that $\nu(i) \succeq_i \mu(i)$ for all $i \in I$ and $\nu(i) \succ_i \mu(i)$ for some $i \in I$.

Model

Definition

An equal-term matching μ is **in the equal-term core**, or, simply **in the core**, if there is no coalition $T \subseteq I$ and a equal-term matching ν such that,

- $h(\nu(i)) \in \{h_j\}_{j \in T}$ for all $i \in T$,
- $\nu(i) \succeq_i \mu(i)$ for all $i \in T$,
- $\nu(i) \succ_i \mu(i)$ for some $i \in T$.

Model

Remark

In a house exchange problem with contract, core may be empty.

i_1	i_2	i_3
$(2, L)$	$(3, S)$	$(1, L)$
$(3, L)$	$(1, L)$	$(2, S)$
$\bar{1}$	$\bar{2}$	$\bar{3}$

There are three efficient matching in this problem.
$(i_1 - 2L, i_2 - 1L, i_3 - \bar{3}); (i_1 - \bar{1}, i_2 - 3S, i_3 - 2S); (i_1 - 3L, i_2 - \bar{2}, i_3 - 1L)$.
The first one is blocked by (i_2, i_3) coalition, second one by (i_3, i_1), and the third one by (i_1, i_2).

Theorem

There exists an efficient, individually rational, equal-term matching in every housing market with contract problem.

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Proof.

There is at least one equal-term and individually rational matching, the initial endowment. Take all equal-term and individually rational matchings. There should be at least one that is not dominated by any other matchings in the set. That matching is the efficient, individually rational, and equal-term matching. □

- We want to find an efficient, individually rational, and equal-term matching.
- However, the next theorem tells that it is not possible to achieve those properties in a strategy-proof way.

Theorem

There is no mechanism that is Pareto efficient, individually rational, equal-term, and strategy proof.

Proof.

Suppose there are two agents with the following preferences.

i_1	i_2
$(2, L)$	$(1, S)$
$(2, S)$	$(1, L)$
$\bar{1}$	$\bar{2}$

There are two efficient and equal-term matchings; $(i_1 - 2L, i_2 - 1L); (i_1 - 2S, i_2 - 1S)$. If the mechanism chooses the first one, then i_2 can manipulate by stating $(1, L)$ is not acceptable for her. If the mechanism chooses the second one, then i_1 can manipulate by stating $(2, S)$ is not acceptable for her. □

How to find an efficient matching?

- Since it is not possible to have an efficient and strategy proof mechanism,
- Focus on efficient mechanism, ignoring strategy proof.

How to find an efficient matching?

- Why drop strategy proof?
- If we were to keep strategy proofness, how far can we reach in efficiency?

Strategy Proof Mechanism

Definition

A mechanism is *Pareto improving from endowment* if it chooses a matching that Pareto dominates initial endowment whenever it's available.

Strategy Proof Mechanism

Theorem

If a mechanism is strategy proof, individually rational, and equal-term, then it cannot be Pareto improving from endowment.

Strategy Proof Mechanism

- Mechanism should choose between L-term and S-term at some point.
- An agent who dislikes it can manipulate by reporting that the contract is unacceptable.
- So we drop the strategy proof, and try to find an efficient, individually rational, and equal-term matching.

Strategy Proof Mechanism

Definition

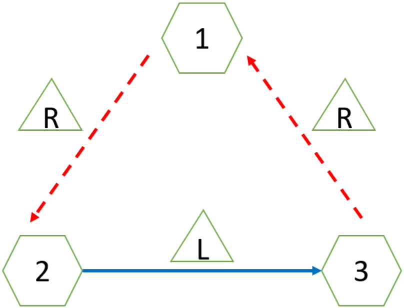
A mechanism is *not manipulable by unacceptable contracts* if, no agent can manipulate by stating her unacceptable contract as acceptable.

Strategy Proof Mechanism

Theorem

Any Pareto improving mechanism that is not manipulable by unacceptable contracts Pareto dominates any strategy proof mechanism. (not only in reported preferences, but in true preferences.)

How to find an efficient matching?



Top trading cycle does not work.

Top Trading Cycle with Counter Offer

- Similar to YRMH-IGYT (Sönmez and Ünver)
- but with potential “Counter Offer”