

# Parking Space Assignment : A Matching Mechanism Design Approach

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## Motivation

In a modern city like Boston, finding a (public) parking space is not an easy task. A driver should cruise to find a spot, sometimes wasting more time than spent on driving. This cruising behavior contributes to the traffic congestion and air pollution, increasing the total cost of parking for all drivers and for the society.

The main research question in this paper is to allocate on-side public parking spaces efficiently while minimizing negative side-effect of cruising-for-parking behavior.

## Model

A **parking slot assignment problem** is a list  $(I, S, \succ_I, D)$  with

$I = \{i_1, \dots, i_n\}$  : a set of drivers at a given point of time.

$S = \{s_1, \dots, s_m\}$  : a set of parking slots at a given point of time.

$\succ_I = (\succ_{i_1}, \dots, \succ_{i_n})$  : a list of drivers' strict preferences over slots.

$D = (\dots, d_{is}, \dots)$  : a list of distances from each driver to each slot, which will serve as priorities of drivers at each slot.

By interpreting the distances as preferences of the slots, we can treat this problem as two-sided matching problem. This is reasonable since, if possible, we want to minimize the total distance traveled by drivers.

$P_S = (P_{s_1}, \dots, P_{s_m})$  : slots' strict preferences over drivers, which satisfy

$$iP_{s_j} i' \text{ iff } d_{is} < d_{i's}$$

## Matching and Stability

The outcome of the parking slot assignment problem is a matching.

A matching  $\mu: I \rightarrow S$  is a function from the set of drivers to the set of the slots such that no slot is assigned to more than one driver.

$\mu(i)$  is a slot that a driver  $i$  is assigned under the matching  $\mu$ , and  $\mu^{-1}(s)$  is a driver that a slot  $s$  is matched to.

▶ A driver-slot pair  $(i, s)$  blocks matching  $\mu$  if either

(i)  $s \succ_i \mu(i)$  and  $iP_{s\mu^{-1}(s)}$

(ii)  $s \succ_i \mu(i)$  and  $\mu^{-1}(s) = \emptyset$

▶ A matching  $\mu$  is **stable** if there is no driver-slot pair  $(i, s)$  that blocks  $\mu$ .

## Cruising Game

In a decentralized parking market, drivers are facing a game situation, namely a **cruising game**, where

- > players are the drivers,  $I$
- > strategy is the set of the slots,  $S$
- > and the outcome is a matching.

## First observation

### Theorem

The set of Nash equilibrium outcomes of the cruising game is equal to the set of stable matchings of the parking slot assignment problem.

## Drivers Proposing Deferred Acceptance (DPDA) Algorithm

**Step 1** : Each driver  $i$  proposes to her 1st choice (among all acceptable choices). Each space  $s$  tentatively holds the closest proposal, if any, and reject the others.

⋮

**Step k** : Any driver who was rejected at step  $k-1$  proposes to the best acceptable space which she hasn't yet made an offer. Each space holds the closest proposal among all the offers including it was holding, and rejects the others. If no rejections occurs, finalize the mechanism and match the "holding" offers.

1. DPDA results in drivers-optimal stable matching.
  2. DPDA is strategy proof for drivers.
- > It is the best for the drivers under the stability constraint.

## In the paper

The model is extended to a **matching with contracts model**, to incorporate

- > endogenous price system
- > resident parking spaces
- > duration terms (short- and long-term)
- > dynamic mechanism

